MTH 3081

Assignment #2: SOLUTIONS

(1) (p. 41) (4). "at least one occurs" = $A \cup B$

"A occurs but B does not occur" = $A \cap B^c$.



$\Rightarrow A \cup B = (A \cap B^c) \cup B$. and those are disjoint

$\Rightarrow P(A \cup B) = P(A \cap B^c) + P(B)$

$0.3 = 0.1 + P(B) \Rightarrow \boxed{P(B) = 0.2}$

(5). $A_1 \cup A_2 \cup A_3$ = "at least one of A_1, A_2, A_3 occurs"

= "at least one 6 shows on the 3 dice".

So $(A_1 \cup A_2 \cup A_3)^c$ = "no 6 shows" = $A_1^c \cap A_2^c \cap A_3^c$.

$\Rightarrow P((A_1 \cup A_2 \cup A_3)^c) = \left(\frac{5}{6}\right)\left(\frac{5}{6}\right)\left(\frac{5}{6}\right) \neq \frac{1}{2} \Rightarrow P(A_1 \cup A_2 \cup A_3) \neq \frac{1}{2}$.

(9). "nobody eliminated on first toss" = "all coins come up same".

Now $P(\text{all coins same}) = P(HHH) + P(TTT) = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$.

So $P(\text{not all coins same}) = \frac{3}{4}$.

Since someone is eliminated unless all coins are same, we get

$P(\text{someone is eliminated}) \leq \frac{3}{4}$.

(14). $P((A \cup B \cup C)^c) = 1 - P(A \cup B \cup C)$.

This is smallest when $P(A \cup B \cup C)$ is largest. But largest value is when A, B, C are disjoint, so

$P(A \cup B \cup C) \leq P(A) + P(B) + P(C) = 0.6$

$\Rightarrow P((A \cup B \cup C)^c) \geq 0.4$.

[2]. $P(A\cup B\cup C) = P((A\cup B)\cup C) = P(A\cup B) + P(C) - P((A\cup B)\cap C)$

$= P(A) + P(B) - P(A\cap B) + P(C)$

$- P((A\cap C)\cup(B\cap C))$

$= P(A) + P(B) + P(C) - P(A\cap B)$

$- P(A\cap C) - P(B\cap C)$

$+ P(A\cap B\cap C)$.

[3]. A: first bet succeeds.
B: second bet succeeds.

Given: $P(A) = 0.7$, $P(B) = 0.4$, $P(A^c\cup B^c) = 0.75$

Want: $P(A\cup B)$.

Now $A^c\cup B^c = (A\cap B)^c \Rightarrow P(A\cap B) = 1 - P((A\cap B)^c) = 0.25$.

so $P(A\cup B) = P(A) + P(B) - P(A\cap B) = 0.7 + 0.4 - 0.25 = 0.85$.

[4] (p. 51) (1). A: sum = 10
B: sum > 8.

$P(A) = P(4,6) + P(5,5) + P(6,4) = \frac{3}{36}$.

$P(B) = P(3,6) + P(4,6) + P(5,6) + P(6,6) + P(6,5) +$ ~~$P(5,4)$~~
$+ P(5,4) + P(5,5)$ ~~$+ P(6,5)$~~

$+ P(6,3) + P(6,4) + P(6,5) = \frac{10}{36}$.

$P(A\cap B) = P(A) = \frac{3}{36}$

$\Rightarrow P(A|B) = \frac{P(A\cap B)}{P(B)} = \frac{3/36}{10/36} = 0.3$

(3)

(2). $P(A|B) + P(B|A) = \frac{P(A \cap B)}{P(B)} + \frac{P(B \cap A)}{P(A)} = 0.75$

$\Rightarrow P(A \cap B) = 0.75 \left(\frac{1}{P(B)} + \frac{1}{P(A)}\right)^{-1} = 0.75\left(\frac{1}{0.4} + \frac{1}{0.2}\right)^{-1} = 0.1$

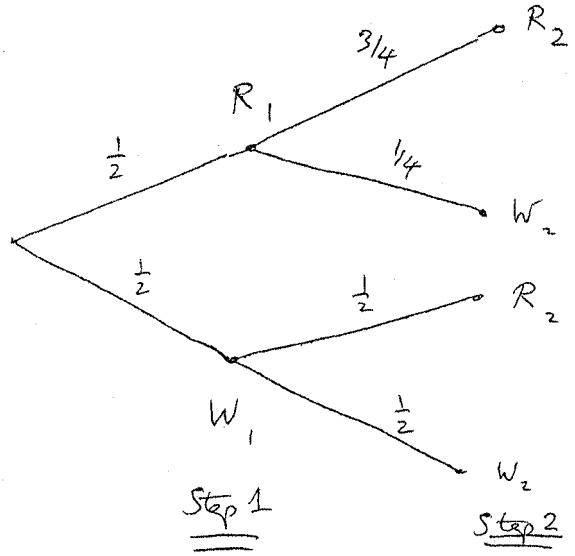
(3). $P(B|A) = \frac{P(B \cap A)}{P(A)}$

$P(A|B) = \frac{P(A \cap B)}{P(B)}$

$\Rightarrow \frac{P(B|A)}{P(A|B)} = \frac{P(B)}{P(A)}$.

$\Rightarrow \frac{P(B|A)}{P(B)} = \frac{P(A|B)}{P(A)}$.

(7)



$P(\text{both Red}) = P(R_2 \cap R_1) = P(R_2|R_1)\,P(R_1) = \left(\frac{3}{4}\right)\left(\frac{1}{2}\right) = \frac{3}{8}$.

(11). Let A : person likes A
B : person likes B

(a) $P(\text{likes neither}) = P(A^c \cap B^c) = 1 - P(A \cup B)$

$= 1 - P(A) - P(B) + P(A \cap B)$

$= 1 - 0.65 - 0.55 + 0.25$

$= 0.05$

(b) $P(\text{likes exactly one}) = P(A) + P(B) - 2P(A \cap B) = 0.7$

(c) $P(\text{likes at least one}) = P(A \cup B) = 0.95$

(d) $P(\text{likes at most one}) = 1 - P(A \cap B) = 0.75$

(e) $P(\text{likes exactly one} \mid \text{likes at least one})$

$= \dfrac{P(\text{likes exactly one})}{P(\text{likes at least one})} = \dfrac{0.7}{0.95}$

(f) $P(\text{like both} \mid \text{like at least one})$

$= \dfrac{P(A \cap B)}{P(A \cup B)} = \dfrac{0.25}{0.95}$

(g) $P(B \mid A^c) = \dfrac{P(B \cap A^c)}{P(A^c)} = \dfrac{0.55 - 0.25}{1 - 0.65} = \dfrac{0.3}{0.35}$

5 (p. 61)

(26) Let H, T be outcome of coin toss.

Let A : event that die (or dice) is 6

$$P(A) = P(A|H)\,P(H) + P(A|T)\,P(T)$$

$$= \left(\frac{1}{6}\right)\left(\frac{1}{2}\right) + \left(\frac{5}{36}\right)\left(\frac{1}{2}\right) \qquad \left(\frac{5}{36} = \text{prob. that sum of two dice is } 6\right)$$

$$= \frac{11}{72}.$$

(34)

$P(\text{seventh chip is Red})$

$= P(\text{first chip is Red})$

$= \frac{40}{100}$